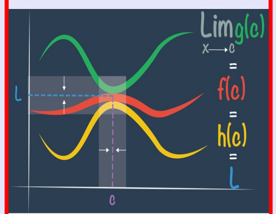
Math 261

Fall 2022

Lecture 39



Feb 19-8:47 AM

$$\frac{d}{dx}\left[\frac{x^{n+1}}{n+1}\right] = \frac{\cancel{(n+1)}}{\cancel{n+1}} x^{n+\cancel{1}-\cancel{1}} = x^n$$

$n \neq -1$

$$\boxed{\int x^n = \frac{x^{n+1}}{n+1} + C}$$

$$\frac{d}{dx}\left[\sin x\right] = \cos x \Rightarrow \boxed{\int \cos x \, dx = \sin x + C}$$

$$\frac{d}{dx}\left[-\cos x\right] = -(-\sin x) = \sin x \Rightarrow \boxed{\int \sin x \, dx = -\cos x + C}$$

$$\frac{d}{dx}\left[\tan x\right] = \sec^2 x \Rightarrow \boxed{\int \sec^2 x \, dx = \tan x + C}$$

$$\frac{d}{dx}\left[\sec x\right] = \sec x \tan x \Rightarrow \boxed{\int \sec x \tan x \, dx = \sec x + C}$$

$$\int c f(x)\,dx = c \int f(x)\,dx$$

$$\int [f(x) + g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$$

Nov 7-8:48 AM

Find $\int \left(\frac{3}{4}x^2 - \frac{4}{5}x^3\right)dx =$

$$\int \frac{3}{4}x^2 dx - \int \frac{4}{5}x^3 dx =$$

$$\frac{3}{4}\int x^2 dx - \frac{4}{5}\int x^3 dx =$$

$$\frac{\cancel{3}}{4} \cdot \frac{x^3}{\cancel{3}} + C_1 - \frac{\cancel{4}}{5} \cdot \frac{x^4}{\cancel{4}} + C_2 =$$

$$\boxed{\frac{1}{4}x^3 - \frac{1}{5}x^4 + C}$$

Nov 7-8:55 AM

Find $\int [6\theta^2 - 4\sin\theta]\,d\theta =$

$$= \frac{6\theta^3}{3} - 4\cdot(-\cos\theta) + C$$

$$= \boxed{2\theta^3 + 4\cos\theta + C}$$

$f''(x) = \frac{3}{\sqrt{x}}$ $f(4) = 20$, $f'(4) = 7$

Find $f(x)$.

$$f'(x) = \int f''(x)\,dx = \int \frac{3}{\sqrt{x}}\,dx = \int 3x^{-1/2}dx$$

$$= 3\cdot\frac{x^{1/2}}{1/2} + C$$

$f'(x) = 6\sqrt{x} + C$

$f'(4) = 6\sqrt{4} + C = 7$

$12 + C = 7$

$\boxed{C = -5}$

$$f'(x) = 6\sqrt{x} - 5$$

$$f(x) = \int f'(x)\,dx$$

$$= \int (6\sqrt{x} - 5)\,dx = \int (6x^{1/2} - 5)\,dx = \frac{6x^{3/2}}{3/2} - 5x + C$$

$$= 4x\sqrt{x} - 5x + C$$

$\boxed{f(x) = 4x\sqrt{x} - 5x + 8}$

$f(4) = 4\cdot4\sqrt{4} - 5(4) + C = 20$

$32 - 20 + C = 20$ $\boxed{C = 8}$

Nov 7-8:58 AM

Definite integral:

If $\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\cdot\Delta x$ exists, then

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\cdot\Delta x = \int_a^b f(x)\,dx$$

And if $\frac{d}{dx}[F(x)] = f(x)$ $\quad = F(x)\Big|_{x=a}^{x=b}$

$$= F(b) - F(a)$$

Note:

If $f(x) \geq 0$ for $[a,b]$, then

$\int_a^b f(x)\,dx$ is the area

under $f(x)$, above x-axis,

from $x=a$ to $x=b$.

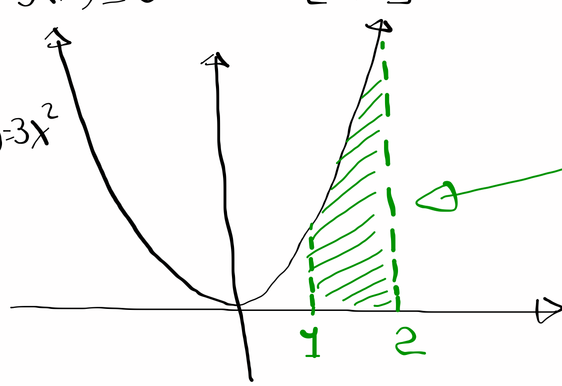
Nov 7-9:06 AM

Evaluate $\int_1^2 \underbrace{3x^2}_{f(x)}\,dx = \cancel{3}\frac{x^3}{\cancel{3}}\Big|_1^2 = \underbrace{x^3}_{F(x)}\Big|_1^2$

$f(x) \geq 0$ for $[1,2]$

$= 2^3 - 1^3 = \boxed{7}$

$y = 3x^2$



Nov 7-9:11 AM

Evaluate $\int_0^3 (x^3 - 6x)\,dx$

$$= \left(\frac{x^4}{4} - \frac{6x^2}{2}\right)\Bigg|_{x=0}^{x=3} = \left(\frac{1}{4}x^4 - 3x^2\right)\Bigg|_0^3$$

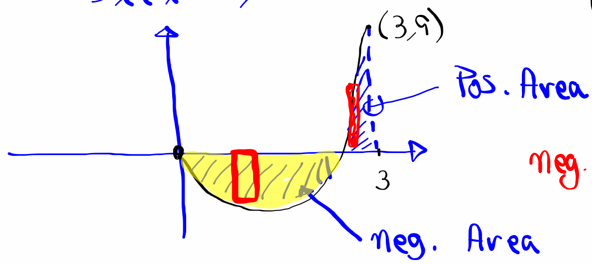
$$= \left[\frac{1}{4}\cdot(3)^4 - 3\cdot 3^2\right] - \left[\cancel{\frac{1}{4}\cdot 0^4 - 3\cdot 0^2}\right]^{\to 0}$$

$$= \frac{81}{4} - 27 = \boxed{-6.75}$$

Can Area be negative? NO

$f(x) = x^3 - 6x$
$= x(x^2 - 6) \to x = \pm\sqrt{6}$

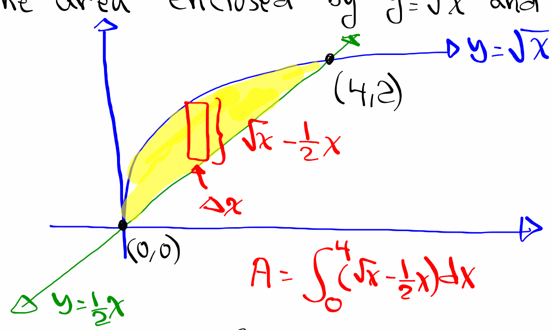
Can definite integral be negative? Yes



neg. Area + Pos. Area = -6.75

Nov 7-9:15 AM

Find the area enclosed by $y = \sqrt{x}$ and $y = \frac{1}{2}x$.



$$A = \int_0^4 \left(\sqrt{x} - \frac{1}{2}x\right)dx$$

$$= \left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{1}{2}\cdot\frac{x^2}{2}\right)\Bigg|_0^4$$

$$= \left(\frac{2}{3}x\sqrt{x} - \frac{1}{4}x^2\right)\Bigg|_0^4$$

$$= \frac{2}{3}\cdot 4\cdot\sqrt{4} - \frac{1}{4}\cdot 4^2 - 0$$

$$= \frac{16}{3} - 4 = \frac{16}{3} - \frac{12}{3} = \boxed{\frac{4}{3}}$$

$\sqrt{x} = \frac{1}{2}x$

$x = \frac{1}{4}x^2$

$4x = x^2$

$x^2 - 4x = 0$

$x(x-4) = 0$

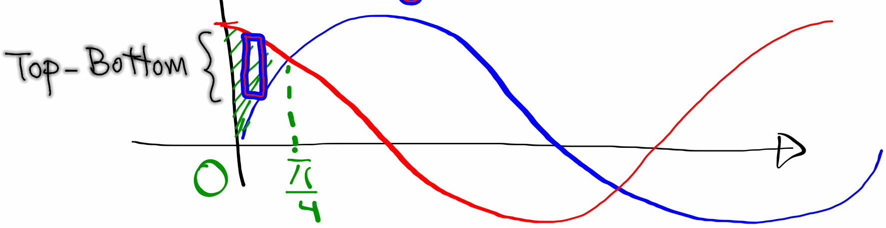
$x = 0 \quad x = 4$

Nov 7-9:23 AM

Find the enclosed area bounded by

$f(x) = \cos x$, $g(x) = \sin x$ for $0 \le x \le \frac{\pi}{4}$

$$A = \int_0^{\pi/4} (\cos x - \sin x)\,dx = \Big[\sin x + \cos x\Big]\Big|_{x=0}^{x=\frac{\pi}{4}}$$

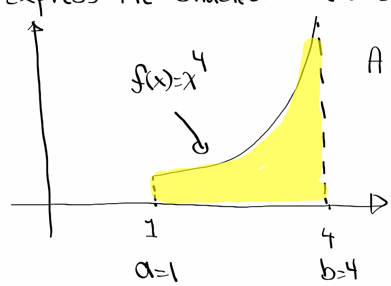


$$= \underbrace{\left(\sin\frac{\pi}{4} + \cos\frac{\pi}{4}\right)}\ - \left(\sin 0 + \cos 0\right) \qquad \sin 0 \to 0, \quad \cos 0 \to 1$$

$$= \boxed{\sqrt{2} - 1}$$

Nov 7-9:32 AM

Express the shaded area below as limits



$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\cdot\Delta x$$

$$\Delta x = \frac{b-a}{n} = \frac{4-1}{n} = \frac{3}{n}$$

$$x_i = a + i\,\Delta x$$
$$= 1 + i\cdot\frac{3}{n} = 1 + \frac{3i}{n}$$

$$f(x_i) = \left(1 + \frac{3i}{n}\right)^4$$

$$\boxed{A = \lim_{n\to\infty} \sum_{i=1}^{n} \left(1 + \frac{3i}{n}\right)^4 \cdot \frac{3}{n}}$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$

$$\sum_{i=1}^{n} i^4 = \frac{1}{4}n^4 + \frac{1}{2}n^3 + \frac{1}{4}n^2$$

$$A = \int_1^4 x^4\,dx = \frac{x^5}{5}\Big|_1^4 = \frac{1}{5}\left[4^5 - 1^5\right] = \boxed{\frac{1023}{5}} \checkmark$$

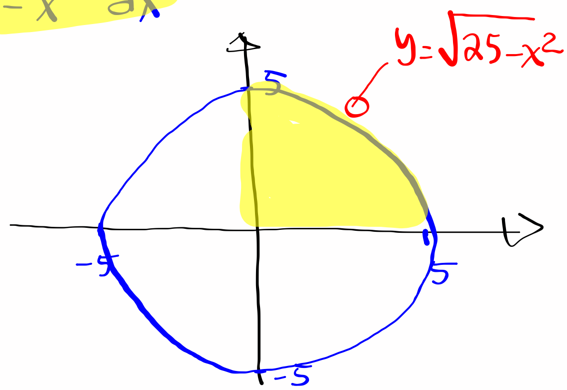
Nov 7-9:38 AM

Evaluate $\int_0^5 \sqrt{25-x^2}\,dx$

$y = \sqrt{25-x^2}$

$y^2 = 25 - x^2$

$x^2 + y^2 = 25$



$= \frac{1}{4}$ Area of Circle $= \frac{1}{4} \cdot \pi r^2$

$= \frac{1}{4} \cdot \pi \cdot 5^2 = \boxed{\frac{25\pi}{4}}$